A Mathematical Modeling on Prey-predator System

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Abstract - **We Seek to propose and analyze a mathematical model in respect with two species Prey-predator system. The existence, uniqueness and boundedness of the solution of the proposed model are discussed. All the equilibrium points are determined. The local stability analysis of them are examined. In this paper are investigated the role of reserved zone on the dynamical behaviour of Prey - predator in the case of when predator species are totally depends on Prey species. The condition for the system to be uniformly persistent have been derived. We shown that the reserve zone has a stabilizing effect on Prey- Predator interactions.**

Key words- **Prey predator, reserved zone, Stability, persistence.**

I. INTRODUCTION

The development of qualitative analysis of ordinary Differential equations is derived to study many problems in mathematical biology. The biosphere is an important zone for biological activities that are mainly responsible for the changes in ecology and environment [1]. The co- existences of interacting biological species has been of great interest in the past few decades and has been studied extensively using mathematical models by several researcher $[2-5]$. Here we assume a habitat where prey and predator species are living together. We divided habitat consisting of two zones : an unreserved zone where prey and predator can move freely and reserved zone where prey can live but predators are not allowed to enter there. We examined of the case when the predator species wholly dependent on the prey in the unreserved zone. We consider the model developed by incorporating and additional equation for predator in the unreserved zone [1].Then we study the co-existence and stability behaviour of predator- prey system in the habitat. Instead of developing explicit formulas for these differential equations, we instead make only qualitative assumption about the form of the equations. We then derive geometric information about the behaviour of solutions of such systems based on these assumption [3].

II. MATHEMATICAL MODEL

At first we consider that the prey species are growing reserved zone as well as in unreserved zone logistically. Let us consider the density of prey species in unreserved zone be $x(t)$ and the density of prey species in reserved zone be $y(t)$. Beside these we consider the density of predator species at any time $t \geq 0$.

Now let ∂_1 be the migration rate coefficient of the prey species from unreserved zone to reserved zone and ∂_2 be the migration rate coefficient of prey species from reserved zone to unreserved zone [3].

Then the dynamical system may be generated by the of ordinary Differential Equations [4], given below

$$\left.\begin{aligned} \frac{dx}{dt} &= w_1 x - K_1 x^2 - \partial_1 x + \partial_2 y - b_1 xz \\ \frac{dy}{dt} &= w_2 y - K_2 y^2 + \partial_1 x - \partial_2 y \\ \frac{dz}{dt} &= \psi(z) - C_0 z \end{aligned}\right\} \text{----------(A)}$$

Where, $x_0 = x(0) \geq 0$

$y_0 = y(0) \geq 0$

$z_0 = z(0) \geq 0$

The parameter in the system of equations of (A) are described as follows;

w_1= Intrinsic growth rate coefficient in unreserved zone
K_1=The intra-specific competitions (between the preys) rate of coefficient for unreserved zone
b_1= The rate of coefficient at which predator removes prey.
w_2= Intrinsic growth rate coefficient in reserved zone.
K_2= The rate of coefficient of intra-specific competition between the preys in reserved zone.
ψ(z)= Growth rate of Predator
C_0= Natural death rate coefficient of predators.

Now using carrying capacities m for the species prey in unreserved zone and n for the species prey in reserved zone [3], we have the transformed form of the system (A) as follows:

$$\left.\begin{aligned} \frac{dx}{dt} &= w_1x\left(1-\frac{x}{m}\right)-\partial_1x+\partial_2y-b_1xz \\ \frac{dy}{dt} &= w_2y\left(1-\frac{y}{n}\right)+\partial_1x-\partial_2y \\ \frac{dz}{dt} &= \psi(z)-c_0z \end{aligned}\right\} \text{------- } (B)$$

Where $\frac{w_1}{m}=k_1$ and $\frac{w_2}{n}=k_2$

When predator species are wholly dependent on prey species:

Let, $\psi(z)=b_0xz$ ------- (i)

From the 3rd equation of the system (B), we have $b_0xz-c_0z=0$ i.e, $x=\frac{C_0}{b_0}$ or, $z=0$

Case 1: When $x=\frac{C_0}{b_0}$, $z>0$

$$\frac{dx}{dt}=0$$

$$\Rightarrow\ w_1x\left(1-\frac{x}{m}\right)-\partial_1x+\partial_2y-b_1xz=0 \text{ -- } (ii)$$

$$\frac{dy}{dt}=0$$

$$\Rightarrow\ w_2y\left(1-\frac{y}{n}\right)+\partial_1x-\partial_2y=0 \text{ ---- } (iii)$$

Putting $x=\frac{C_0}{b_0}$ in (iii) , we get

$$w_2y\left(1-\frac{y}{n}\right)+\partial_1\frac{C_0}{b_0}-\partial_2y=0$$

Or, $b_0w_2y^2+(\partial_2-w_2)b_0ny-c_0n\partial_1=0$

$$\therefore y=\frac{1}{2b_0w_2}\left[(w_2-\partial_2)+\sqrt{(w_2-\partial_2)^2+4b_0w_2c_0n\partial_1}\right]$$

(for positive value)

Now if we put the value of x and y in (ii),

We have,

$$w_1\left(\frac{C_0}{b_0}\right)\left(1-\frac{C_0}{b_0m}\right)-\partial_1\left(\frac{C_0}{b_0}\right)+\partial_2\frac{1}{2b_0w_2}\left[(w_2-\partial_2)+\sqrt{(w_2-\partial_2)^2+4b_0w_2c_0n\partial_1}\right]-b_1\left(\frac{C_0}{b_0}\right)z=0$$

After calculating,

$$z=\frac{b_0}{C_0b_1}\left[\partial_2y+(w_1-\partial_1)\frac{c_0}{b_0}-\frac{w_1c_0}{mb_0{}^2}\right]$$

For positive solution, $z>0$

i.e, $\partial_2y+(w_1-\partial_1)\frac{C_0}{b_0}>\frac{w_1C_0}{mb_0{}^2}$

i.e, $\frac{1}{2b_0w_2}\left[(w_2-\partial_2)+\sqrt{(w_2-\partial_2)^2+4b_0w_2c_0n\partial_1}\right]+(w_1-\partial_1)\frac{C_0}{b_0}>\frac{w_1C_0}{mb_0{}^2}$

Which can give the initial value of carrying capacity of the free access zone for the survival predators.

∴ In this case the equilibrium point is (x^*,y^*,z^*)

Where $x^*=\frac{C_0}{b_0}$

$$y^*=\frac{1}{2b_0w_2}\left[(w_2-\partial_2)+\sqrt{(w_2-\partial_2)^2+4b_0w_2c_0n\,\partial_1}\right]$$

$$z^*=\frac{b_0}{C_0b_1}\left[\frac{\partial_2}{2b_0w_2}\left\{(w_2-\partial_2)+\sqrt{(w_2-\partial_2)^2+4b_0w_2c_0n\,\partial_1}\right\}+(w_1-\partial_1)\frac{C_0}{b_0}-\frac{w_1C_0}{mb_0{}^2}\right]$$

Case 2: When $z = 0$

Subcase (i)*:*

When $z = 0$, we can have $x = 0$, $y = 0$

$\therefore (x^*, y^*, z^*) = (0,0,0)$ is also an equilibrium point.

Subcase (ii): Again when $z = 0$, (ii) becomes

$$w_1 x\left(1-\frac{x}{m}\right) - \partial_1 x + \partial_2 y = 0 \quad \text{--------} \quad (iv)$$

$$\therefore y = \frac{\partial_1}{\partial_2}x - \frac{w_1}{\partial_2}x\left(1-\frac{x}{m}\right) \quad \text{------} \quad (v)$$

Putting the value of y in (iv) , we get (after computing)

$$\left(\frac{w_1{}^2 w_2}{n\,\partial_2{}^2 m^2}\right)x^3 + \left\{\frac{-2w_1 w_2(w_1-\partial_1)}{mn\,\partial_2{}^2}\right\}x^2 + \left\{\frac{w_2(w_1-\partial_1)^2}{n\,\partial_2{}^2} - \frac{(w_2-\partial_2)w_1}{\partial_2 m}\right\}x + \left(\frac{w_1 w_2 - \partial_1 w_2 - w_1\,\partial_2}{\partial_2}\right) = 0$$

For unique positive solution

$$\frac{w_2(w_1-\partial_1)^2}{n\,\partial_2{}^2} - \frac{(w_2-\partial_2)w_1}{\partial_2 m} < 0$$

$$\Rightarrow \frac{w_2(w_1-\partial_1)^2}{n\partial_2} < \frac{(w_2-\partial_2)w_1}{m}$$

There is no migration of the prey species from reserved zone to unreserved zone $(\partial_2 = 0)$ and $(w_1 - \partial_1) < 0$ i.e, $\frac{dx}{dt} < 0$

Similarly there is no migration from the prey species from unreserved zone to reserved zone $(\partial_1 = 0)$ and $(w_2 - \partial_2) < 0$ i.e. $\frac{dy}{dt} < 0$.

So our assumption is that $w_1 > \partial_1$ and $w_2 > \partial_2$

$$\therefore x^* > \frac{m}{w_1}(w_1 - \partial_1)$$

Putting the value of x^* in (v) , we can get the value of y^* as $y^* = \frac{1}{\partial_2}\left[\frac{w_1(x^*)^2}{m} - (w_1-\partial_1)x^*\right]$

Where $x^* > \frac{m}{w_1}(w_1 - \partial_1)$

III. STABILITY ANALYSIS

(i) (0,0,0) is a saddle point.

(ii) $(x^*, y^*, 0)$ is a saddle point if $b_0 x^* > C_0$

(iii) (x^*, y^*, z^*) is asymptotically stable if $b_0 x^* < C_0$.

IV. CONCLUSION

In this paper a mathematical model has been analyzed in the case when predator species are wholly dependent on the prey. In the absence of predator , the density of prey is maximum is reserved as well as unreserved zone.The cumulative density of prey reduces when predator species wholly depends on prey species. This study suggests that the role of reserved zone is an important integrating concept in ecology and evolution.

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